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ABSTRACT

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MULTIVARIATE INTRACLASS CORRELATION

Abstract

This paper is an explication of a statistical model which will permit an interpretable intraclass correlation coefficient that is negative, and a generalized extension of that model to cover a multivariate problem. The methodological problem has its practical roots in an attempt to find a statistic which could indicate the degree of similarity or dissimilarity between members of sociometric, reciprocal choice dyads. The intraclass correlation coefficient is a coefficient designed to show similarity; our problem was to extend the underlying model to allow interpretation of a coefficient indicating dissimilarity, and to simultaneously consider more than one variable.

MULTIVARIATE INTRACLASS CORRELATION

An important theoretical and empirical issue in research on interpersonal relations revolves around the nature of attraction between two persons who either select or associate with each other in an activity or relationship. One method of investigating the nature of this attraction is by studying the relationships between personalities of persons who are members of particular dyadic associations. The two most common types of association which have been studies this way are marriage and friendship, the latter operationalized as mutual choice on a sociometric questionnaire.

In most of the research in this area investigators have analyzed the composition of a set of dyads with regard to the similarity or dissimilarity of members on particular traits, on a series of traits, and in a few instances, have analyzed the relationships across traits between members. These studies have generally been framed within the context of supporting either similarity or complementarity as the one inclusive principle of choice or association.

With regard to such traits as achievement, I.Q., socioeconomic class, age, class in college, grade point average, there has been found low positive correlation indicating similarity between persons who choose each other on sociometric questionnaires (Richardson, 1939; Morton, 1959). These traits we call sociological descriptive variables in that they are hierarchies used to define one's place in a group, organization, or society. A person at a particular level in the hierarchy is likely to come into contact with others in the same level; thus one's contacts limits his selection of associates to others who are similar to himself on these variables.

However, another set of studies have investigated similarity and complementarity of members of reciprocal choice dyads on psychologically more dynamic traits of personality as represented in need states. With the exception of a series of studies by Winch (1955a, 1955b)(Winch, Ktsanes and Ktsanes, 1954, 1955) who studied psychological complementation in twenty-five pairs of newlyweds, these studies have shown either no relationship (Fiedler, Warrington, Blaisdell, 1952; Izard, 1963; Miller, Campbell, Twedt, and Carroll, 1966; Reilly, Commins, and Stefic,

1960) or trends toward similarity (Izard, 1960) on these need states between persons in reciprocal choice dyads.

The majority of these studies have used as personality traits scores on the Edwards (1954) Personal Preference Schedules, a personality inventory that measures fifteen needs derived from Murrary (1938).

Recent studies involving reciprocal sociometric choice (Miller, et al., 1966; Izard, 1960, 1963; Reilly, et al., 1960) have used as subjects young adults or late adolescents. The majority of the studies considered only one criterion of choice, that being friendship, without regard to particular function or purpose of the friendship. Most of these studies have assumed that the similarity or complementarity principle should apply to all aspects of personality irrespective of the function or context of choice; and that all traits of personality should be salient in choice. The evidence from these studies doesn't support such assumptions.

We suggest that, in order to find similarity or dissimilarity even on single traits of personality, investigators will have to consider such factors as the purpose of choice and the context of choice. For example, the factors that one might consider in choosing another to co-participate in a work activity might be quite different from those one would consider when choosing another for a social activity. One would not expect all aspects of personality to be equally potent in both choice situations.

In addition to the conceptual problems found in the study of interpersonal attraction, there is the problem of finding clear and systematic statistical methodologies for indicating the similarity or dissimilarity of dyad members on particular traits of personality, and there is the

problem of finding a statistic which indicates the relationship across traits between members. Haggard (1958) and Morton (1959) both suggest that the intraclass correlation would be the appropriate statistic to indicate similarity or dissimilarity on a trait between members of a set of dyads. Izard used this index in his studies (1960, 1963). However, neither Morton (1959), Haggard (1958), Izard (1960), Fisher (1948), or Snedecor (1946) which are referred to by the former, suggest an underlying statistical model that would allow for a negative parameter value.

The Intraclass Correlation

In a short history of the origin of the intraclass correlation index, Haggard (1958) reports that in the early part of the present century a problem for research in the biological sciences was to find the degree of similarity between siblings on various variables. Pearson (1901) suggested that the degree of similarity between siblings on the variable, height, for example, could be derived from a symmetrical correlation table in which the scores of siblings on height would be entered twice in reverse order. The product-moment correlation derived from such a table was called an intraclass correlation coefficient. This is the statisitc used in Reilly, et al. (1960) and is sometimes called the Pearson product moment correlation for interchangeable variable. The necessity for such a procedure derived from the fact the investigator was interested in the relationship of height between siblings in general, not between older and younger, or heavier and lighter siblings. Thus, because there was no logical way of ordering the siblings' scores to be correlated, both scores

were entered in a table twice, using both orders. There were instances when there were more than two siblings in a family. In such cases the acts of siblings were combined in all possible sets of two siblings and the scores of each of the sets of pairs were entered into the symmetrical correlation table in its two orderings. Needless to say, this could be a laborious task for the researcher, particularly when the sets of persons to be correlated amounted to more than two.

In order to overcome this inconvenient process, Harris (1913) developed a new method for estimating the intraclass correlation coefficient. Harris realized that the variance among classes (sets of siblings) could be separated from the total variance (among all individuals); and that the coefficient of intraclass correlation could be defined as the ration of two variances, the between class variance to the total variance. In addition, he realized that these variances could be estimated from two distributions, the distribution of class means and the distribution of the total set of observations.

Fisher, recognizing that unbiased estimates of the variances may be obtained by using the number of degrees of freedom rather than the number of cases in the sample, devised a better estimate of the coefficient of intraclass correlations based upon the mean squares of the analysis of variance table. It is written

$$\hat{\rho}_1 = \frac{MS_c - MS_w}{MS_c + (k-1)MS_w}$$

where $\hat{\rho}_1$ is estimated intraclass correlation coefficient, MS_c is the mean square among classes, MS_w is the mean square among individuals,

and $\underline{k}$ is the number of individuals within a class.

The parameter to be estimated can be defined as:

$$\rho_1 = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_c^2}$$

in which ρ_1 is the intraclass correlation, and σ_c^2 and σ_e^2 are components of variance between and within classes respectively. However, even though the variance component estimates are unbiased, it must be kept in mind that a ratio of these estimates is not necessarily unbiased.

This latter formula is a basic definition of intraclass correlation. It should be noted that the former formula, the estimator, if used for data containing sets of classes containing two members can yield coefficients from -1 to +1; whereas the second formula which is the definition of the intraclass correlation parameter can yield only positive values. When we use the estimator and obtain a +1 value, this means that given a set of classes containing two members each, the variation between members within each class would be zero, whereas there would be variation between class means. If we obtained a -1 value it would signify that all of the classes would have the same mean, but the members vary within classes. However, a problem arises when one tries to interpret the meaning of the latter statistic, since the statistical model underlying the intraclass correlation admits only positive parameter values. This is so even though the test of significance for a negative value, an inverted F-test, can demonstrate a negative value to be significant.

We shall undertake a new development of the model that allows parameters to take on negative values. We shall do this by showing the classical development of the model and modifying this development to admit greater generality.

The following is a development of the classical intraclass correlation model, where the parameter ρ is defined to be a ratio of variances.

Let

$$Y_{ij} = \mu + \gamma_i + \gamma_{ij} \quad ; i = 1, \ldots, n, j = 1, \ldots, n$$

where the classification indexed by j is nested within the classification indexed by i. If we impose the following parameter definitions on the model:

$$\sigma_\beta^2 = \mathrm{Var}(\beta_i), \ \sigma_\gamma^2 = \mathrm{Var}(\gamma_{ij}), \ \mathrm{Cov}(\beta_i, \gamma_{ij}) = 0$$

together with the usual additional independence assumptions, then we may define:

$$\rho = \frac{\mathrm{Cov}(Y_{ij}, Y_{ij'})}{\sqrt{\mathrm{Var}(Y_{ij})\mathrm{Var}(Y_{ij'})}} = \frac{\sigma_\beta^2}{\sigma_\beta^2 + \sigma_\gamma^2}$$

Now, using conventional ANOVA notation where MS_β and MS_γ refer to the mean squares between and within classes respectively, we may define:

$$MS_\beta = \frac{\sum_{i=1}^{m} (\bar{Y}i. - \bar{Y}..)^2}{n - 1}$$

and after application of the expectation operator and some algebra, we find that

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$$E[MS_\beta] = m\sigma_\beta^2 + \sigma_\gamma^2 \quad .$$

Similarly,

$$MS_\gamma = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}(Y_{ij} - Y_{i.})^2}{n(m-1)} \quad \text{and}$$

$$E[MS_\gamma] = \sigma_\gamma^2$$

This leads us to the standard estimator of the intraclass correlation:

$$\beta = \frac{MS_\beta - MS_\gamma}{MS_\beta + (m-1)MS_\gamma} \quad \text{since } E[MS_\beta - MS_\gamma] = m\sigma_\beta^2$$

and

$$E[MS_\beta + (m-1)MS_\gamma] = m(\sigma_\beta^2 + \sigma_\gamma^2) \quad .$$

Again it should be noted that this is a biased estimate, since the ratio of unbiased estimates is not necessarily unbiased. In addition, the parameter ρ may take on values in the range: $0 \leq \rho \leq 1$, since $\sigma_\beta^2 \geq 0$; while the estimate β may take on values in the range: $-1/(m-1) \leq \rho \leq 1$. Note that $-1/(m-1) = -1$, when $m = 2$.

Suppose we attempt to redefine the model so that the values which the parameter may take on match the values which the estimator may assume. Let

$$Y_{ij} = \mu + \alpha_{ij} \quad , \; i = 1, \ldots, n \; ; \; j = 1, \ldots, m,$$

where we impose the same nesting constraints as above.

Define: $\sigma_\alpha^2 = \mathrm{Var}(\alpha_{ij})$ and ρ by the following: $\rho\sigma_\alpha^2 = \mathrm{Cov}(\alpha_{ij}\alpha_{ij'})$.

Then, by algebraic manipulation $E[MS_\beta] = \sigma_\alpha^2[1 + (m + 1)\rho]$ and

$$E[MS_\gamma] = \sigma_\alpha^2(1 - \rho) \quad .$$

Looking at the previous estimator, we find that $E[MS_\beta - MS_\gamma] = m\rho\sigma_\alpha^2$ and $E[MS_\beta + (m - 1)MS_\gamma] = m\sigma_\alpha^2$ reaching the same justification for β as before. The restriction $-1/(m-1) \leq \rho \leq 1$ is now reasonable and it matches the restriction on β .

Another model for the nested classification (McHugh and Mielke, 1968) has recently been proposed. This model also allows valid negative expectations of the difference between the mean squares among and within classes and is not easily adapted to the notion of intraclass correlation.

The model proposed here permits an interpretation of a negative intraclass correlation coefficient. The intraclass correlation coefficient answers the question: Are members of the same dyad alike or dissimilar on this variable, or set of variables? However, the investigator of similarities between members of the same dyad on a particular variable is sometimes interested in the relationship between variables, across persons within dyads. It is not enough to know that members of the same dyad are alike or unlike on a particular variable. It would also be useful to know how their choices were structured across variables. For example, taking the case of the variables A and B , the intraclass correlation would tell how members of the same dyad are similar or dissimilar on variable B , but this would not tell us if there was a choice structure across variables. It might be that there is no structural determinancy on the variables A or B separately, but the choices of the two members of the dyad might be

partially determined by an interactive effect between A and B across members. One member's score on variable A might be related to the other member's score on variable B . The reciprocal relationship, of course, would hold. If such were the case, rather than being led to the conclusion that variable A or variable B , because of a .00 intra-class correlation on each variable respectively, has no influence on choice behavior. This latter interpretation, of course, would depend upon the relationships of these two variables within dyads in general. Another way to look at the same phenomenon would be to conceive of the relationship across variables within individuals. For example, if we know that variables A and B are highly related within individuals, what is the homogeneity of dyads with respect to this relationship?

This problem, although recognized in the literature in terms of statistical methodology, still remains unsolved. Indeed many studies of marriage dyads (Murstein, 1961; Katz, Glucksberg, and Krauss, 1960; Byrne and Baylock, 1963) either ignore it or deal with it by use of frequency counts and chi-square analysis. Izard (1960) by a method of deductive reasoning in which he compares individual intraclass correlations concludes that there are no cross traits relationships. Reilly, et al., and Miller, et al., although they report correlations across traits are unclear as to what type of correlation they used.

Our second methodological problem is to derive a multivariate version of the intraclass correlation model which would allow us to observe the relationships across variables between members of sociometric dyads. This

model would allow us to answer the additional question, what is the relationship across variables between members?

At this point it is appropriate to build on the second version of the intraclass correlation model developed above. Suppose we have the same situation as before except that each unit is measured on more than one variable. Let

$$Y_{ijk} = \mu_k + \alpha_{ijk} \; ; \; i = 1, \ldots, n, \; j = 1, \ldots, m, \; k = 1, \ldots, g \; .$$

Where, i indexes groups, j indexes units nested within groups and k indexes variables.

We may define six parameters for each pair of variables in the following way:

1) $\sigma_k^{2'} = \text{Var}\ (\alpha_{ijk})$

2) $\rho_k \sigma_k^2 = \text{Cov}\ (\alpha_{ijk}, \alpha_{ij'k})$

3) $\sigma_{kk'} = \text{Cov}\ (\alpha_{ijk}, \alpha_{ijk'})$

4) $\rho_{kk'}\sigma_{kk'} = \text{Cov}\ (\alpha_{ijk}, \alpha_{ij'k'})$

Taking the following classical definitions:

1) $MS_{\beta_k} = \dfrac{m \sum_{i=1}^{n} (\bar{Y}_{i.k} - \bar{Y}_{..k})^2}{n - 1}$

2) $MS_{\gamma_k} = \dfrac{\sum_{i=1}^{n} \sum_{j=1}^{m} (Y_{ijk} - \bar{Y}_{i.k})}{n(m - 1)}$

3) $MCP_{\beta_{kk'}} = \dfrac{M \sum_{i=1}^{n} (\bar{Y}_{i.k} - \bar{Y}_{..k})(\bar{Y}_{i.k'} - \bar{Y}_{..k'})}{n - 1}$

4) $MCP_{\gamma_{kk'}} = \dfrac{\sum_{i=1}^{n} \sum_{j=1}^{m} (Y_{ijk} - \bar{Y}_{i.k})(Y_{ijk'} - \bar{Y}_{i.k'})}{n(m - 1)}$

We may proceed to take expectations:

$$1)\quad E[MS_{\beta_k}] = \sigma_k^2[1 + (m - 1)\rho_k]$$

$$2)\quad E[MS_{\gamma_k}] = \sigma_k^2(1 - \rho_{\cdot})$$

$$3)\quad E[MCP_{\beta_{kk'}}] = \sigma_{kk'}[1 + (m - 1)\rho_{kk'}]$$

$$4)\quad E[MCP_{\gamma_{kk'}}] = \sigma_{kk'}(1 - \rho_{kk'}) \quad .$$

TABLE I

Matrix A

Between Group Covariance Matrix

Variable	1	2
1	$\sigma_1^2(1 + \rho_1)$	$\sigma_{12}(1 + \rho_{12})$
2	$\sigma_{12}(1 + \rho_{12})$	$\sigma_2^2(1 + \rho_2)$

TABLE II

Matrix B

Within Group Covariance Matrix

Variable	1	2
1	$\sigma_1^2(1 - \rho_1)$	$\sigma_{12}(1 - \rho_{12})$
2	$\sigma_{12}(1 - \rho_{12})$	$\sigma_2^2(1 - \rho_2)$

Now, we note that it is reasonable to define our estimate of ρ_k and $\rho_{kk'}$, in the following way:

$$1)\quad \delta_k = \frac{MS_{\beta_k} - MS_{\gamma_k}}{MS_{\beta_k} + (m-1)MS_{\gamma_k}}$$

and

$$2)\quad \delta_{kk'} = \frac{MCP_{\beta_{kk'}} - MCP_{\gamma_{kk'}}}{MCP_{\beta_{kk'}} + (m-1)MCP_{\gamma_{kk'}}}$$

since

$$1)\quad E[(MS_{\beta_k} - MS_{\gamma_k})/m] = \rho_k\sigma_k^2$$

$$2)\quad E[(MCP_{\beta_{kk'}} - MCP_{\gamma_{kk'}})/m] = \rho_{kk'}\sigma_{kk'}$$

$$3)\quad E[(MS_{\beta_k} + (m-1)MS_{\gamma_k})/m] = \sigma_k^2$$

$$4)\quad E[(MCP_{\beta_{kk'}} + (m-1)MCP_{\gamma_{kk'}})/m] = \sigma_{kk'}$$

If we use these relations to set up the expected values of two matrices: (1) The Between Covariance Component Matrix defined by 1) and 2) Table III), and 2) The Total Covariance Matrix defined by 3) and 4) Table IV), we may note something interesting. The Total Covariance. Matrix is the same as that which would have been obtained by disregarding the group structure of the data. That is, by rescaling it with the reciprocals of the standard deviations, the raw-between variable correlations would be obtained.

TABLE III

Matrix C

Between Covariance Component Matrix

Variable	1	2
1	$\rho_1\sigma_1^2$	$\rho_{12}\sigma_{12}$
2	$\rho_{12}\sigma_{12}$	$\rho_2\sigma_2^2$

TABLE IV

Matrix D

Total Covariance Matrix

Variable	1	2
1	σ_1^2	σ_{12}
2	σ_{12}	σ_2^2

If we form a new matrix by taking the element by element ratios of the parameter matrices represented in Tables III and IV we obtain the matrix given in Table V. A similar matrix formed using the sample values has the traditional estimates of the intraclass correlations on the diagonals and our bivariate generalizations of these on the off-diagonals. The population matrix may be termed the multivariate intraclass correlation matrix.

TABLE V

Matrix E

Multivariate Intraclass Correlation Matrix

Variable	1	2
1	ρ_1	ρ_{12}
2	ρ_{12}	ρ_2

It should be noted that the off-diagonal elements of this matrix sometimes may be poorly estimated. The denominator of the ratio for the off-diagonal elements is the sum of $MCP_{\beta_{kk'}}$ and $(m-1)MCP_{\gamma_{kk'}}$. If $m = 2$ and $MCP_{\gamma_{kk'}}$, and $MCP_{\gamma_{kk'}}$ have approximately the same absolute value and opposite sign, the ratio may become larger than one. This will occur when $\sigma_{kk'}$, the covariance of variables k and k', is very small. This produces a poorly conditioned estimate of $\rho_{kk'}$.

As can be seen this covariance derivation yields five matrices--A through E (Tables I through V respectively). Matrices A and B are the Between Group Variance Covariance and the Within Group Variance Covariance matrices, respectively. Matrix D (Total Covariance Matrix) which we shall call Relationship Between Variables shows the covariance between two variables in the off-diagonal cells. From these covariances can be derived the equivalent of a product-moment correlation between

variables within individuals. Matrix C is the Between Covariance Component Matrix, which we shall call Relationship Across Variables and People, shows the relationship between variable 1 for member A, and variable 2 for member B of a dyad. Matrix E (Multivariate Intraclass Correlation Matrix) has on its diagonal the classical intraclass correlation of co-choosers on each of the variables. The off-diagonals on this table exhibit the multivariate similarity or dissimilarity of members of a dyad. We shall say that this matrix exhibits similarity across variables between dyad members. A positive sign on this matrix indicates similarity between members, whereas a negative sign indicates dissimilarity.

From a substantive point of view a positive or negative sign on the diagonal is relatively easy to understand. If we take as a hypothetical case the variables A and B, a high positive sign on the diagonal for A would mean high A people choose high A people, while low A people choose low A people. If the relationship were negative, it would mean low A people choose high A people, whereas high A people choose low A people. The same relationships hold, of course, for a positive or negative sign on the variable for B. But what about a high positive or negative sign in the cell that intersects variables A and B? A positive or negative sign here means only that the two choosers are similar or dissimilar to each other with respect to the relationship of A and B within each. In order to interpret the meaning of a positive or negative sign here, we have to refer to covariance between these two variables in Matrix D. Thus, if there is a high negative relationship

between A and B for individuals, a high positive sign in Matrix E would mean this relationship is reinforced across individuals and that high A in one member corresponds to low B in the other. The total degree of this relationship will be found in Matrix C which shows the relationship across people across variables. The reader can see four logical possibilities (non-zero) which can be obtained across people and across variables which can be observed in matrices C, D and E. These are presented in the chart below, the third tier of which we have just described.

LOGICAL POSSIBILITIES OF RELATIONSHIPS BETWEEN CO-CHOOSERS IN RECIPROCAL DYADS ACROSS TWO VARIABLES

Chart A

	MATRIX (D) Relationship Between Variables	MATRIX (E) Similarity Between People	MATRIX (C) Relationships Across People and Variables
I.	+1 (HA HB) and (LA LB)	+1 (HA HB) ↔ (HA HB) and (LA LB) ↔ (LA LB)	+1
II.	+1 (HA HB) and (LA LB)	-1 (HA HB) ↔ (LA LB) and (LA LB) ↔ (HA HB)	-1
III.	-1 (HA LB) and (LA HB)	+1 (HA LB) ↔ (HA LB) and (LA HB) ↔ (LA HB)	-1
IV.	-1 (HA LB) and (LA HB)	-1 (HA LB) ↔ (LA HB) and (LA HB) ↔ (HA LB)	+1

Looking at Chart A we see three columns, each column representing respectively matrices D, E, and C. In the first column, which represents the relationships between two variables, we see two possibilities, a high positive correlation or a high negative correlation. (We will for the purposes of economy eliminate the .00 correlation.) Under each of these possibilities within parantheses are the possible profiles of people in the population given such a correlation. In the second column which represents the similarity between two persons who choose each other with respect to the relationship between two variables within each, we see four possibilities. Again within each parenthesis is a profile of an individual, on each line two individuals are connected by the symbol ↔ meaning they have chosen each other. Thus looking at the top cell in column two, we see that a +1.00 means: given that there is a positive relationship within individuals of A with B, a particular dyad in this population contains two persons who are both high on A and high on B, or are both low on A and low on B. In column three of Chart A is represented the relationship between individuals across variables. A +1.00 under this column would mean that if one of the individuals is high on B, the other is high on A; or if one is low on B, the other is low on A. Whereas, if a -1.00 shows up in column three, it means that if one is high on B, the other is low on A; or if one is low on B, the other is high on A. One can readily see this if he refers to the dyadic profiles represented in the second column. The relationship in column three should represent the relationship between A in the first parantheses and B in the second parentheses. The same relationship, of course, also holds for B in the first parentheses and A in the second parentheses.

An Example

In order to demonstrate the usefulness of the two models, the first allowing for an interpretation of a negative intraclass correlation coefficient, and the second being a generalized multivariate version of the former, we shall present a summary chart of data collected at The University of Chicago in a study directed at the investigation of the emotional dynamics of reciprocal sociometric choice behavior.

The study is used only as an example of the use of these techniques and not for substantive purposes.

In this study we were interested in effects of situational conditions upon the reciprocal sociometric choice behavior of persons characterized according to their typical response pattern to stress in an interpersonal situation. We were particularly interested in the choice behavior of persons who are characterized as Dependent, meaning that they generally deal with stress in interpersonal situations by calling on or seeking authority to deal with the stress; and in persons characterized as Fight, meaning persons who meet stress by hostile aggressive attack on others or the problematic situation. We were interested in whether there would be differences in the types of persons chosen given a difference in the criterion for choice.

One hundred and fifteen children in two fifth and two sixth grade classes were administered a sociometric questionnaire in which they were to select three persons with whom they would like to work on a social studies committee, and three persons with whom they would like to go to the movies. Each of the children was administered a questionnaire in

which the child was asked to choose between alternative emotional responses in meeting an interpersonal problematic situation. This questionnaire yielded five scores, two of which were Fight and Dependency.

From the responses to the sociometric questionnaire two sets of dyads were derived. The first set we shall call work dyads in the sense that they were formed in response to questions about work in social studies. There were 17 dyads in this set. All the dyads were independent of each other in membership (i.e., no two dyads had members in common), however, one of the dyads occurred in response to the movie question. The 21 dyads were independent in membership, and one of these dyads was formed in response to the social studies committee question. Below in Tables VI and VII are the matrices A through E, for each of these sets of dyads on the variables Fight and Dependency.

TABLE VI

Relationships Between Members of Reciprocal Choice Dyads in a Work Situation on the Variables of Fight and Dependency

Matrix A
Between Group Covariance

	F	D
F	14.769	
D	-1.499	8.724

Matrix B
Within Group Covariance

	F	D
F	9.411	
D	-3.676	5.294

Matrix C
Between Covariance Component

	F	D
F	2.679	
D	-2.411	1.715

Matrix D
Total Covariance

	F	D
F	12.090	
D	-5.937	7.009

Matrix E
Multivariate Intraclass Correlation Matrix

	F	D
F	.221	
D	.406	.244

TABLE VII

Relationships Between Members of Reciprocal Choice Dyads in a Play Situation on the Variables of Fight and Dependency

Matrix A
Between Group Covariance

	F	D
F	8.740	
D	.539	2.493

Matrix B
Within Group Covariance

	F	D
F	14.404	
D	-6.523	8.357

Matrix C
Between Covariance Component

	F	D
F	-2.832	
D	3.531	-2.932

Matrix D
Total Covariance

	F	D
F	11.572	
D	-2.992	5.425

Matrix E
Multivariate Intraclass Correlation Matrix

	F	D
F	-.244	
D	-1.180*	-.540

Looking at Table VI we see in Matrix D that within individuals Fight and Dependency are negatively related. This relationship also holds up in Table VII. Looking at Table VI we see that the intraclass correlation coefficients for both Fight and Dependency are positive, although only moderately so. This would seem to indicate that Fight people choose others who are similar to themselves on Fight, that Dependency people choose other persons who are similar to themselves on Dependency. We see in the .406 in the off-diagonal that persons choose others who are similar to themselves in the relationship of Fight

*The reader may refer back to the explanation of Table V, Matrix E, if there is confusion about the range of values the off-diagonal elements can assume.

to Dependency. This in turn means that if a person is high on Fight, he will choose someone low on Dependency, and be chosen in return by such a person.

In contrast to the pattern in Table VI, we see in Table VII a pattern of relationships directly in the reverse, with the exception of the covariance of Fight and Dependency within individuals. Indeed, in Table VII, Matrix E, Fight persons reject others who are similar to themselves on Fight, and likewise choose persons who are dissimilar to themselves on Dependency. The -1.180 in the diagonal of this Matrix means that persons choose others who are dissimilar to themselves with respect to the relationships of Fight and Dependency persons, and are chosen in return by such persons. Likewise, low Fight people choose low Dependent and are chosen in return by these persons. Note that the D Matrices in Table VI and VII are estimates of the same population Matrices except for selection biases.

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